THE SCOTS COLLEGE

2006

TRIAL H.S.C. EXAMINATION

Mathematics

Time Allowed: 3 hours

Instructions

- Show ALL necessary working.
- Approved calculators may be used.
- All questions are of equal marks.
- **Begin each question on a new page.**

STUDENTS ARE ADVISED THAT THIS IS A TRIAL EXAMINATION ONLY AND CANNOT IN ANY WAY GUARANTEE THE CONTENT OR THE FORMAT OF THE HIGHER SCHOOL CERTIFICATE EXAMINATION

Question 1 **(Begin a new Booklet)**

a) Evaluate, to 2 significant figures, $\sqrt[3]{\dfrac{(9.2)^2}{\pi}}$. (1)

b) Simplify fully $\dfrac{6a^2 - 2ab}{9a^2 - b^2}$ (2)

c) Solve for x: $18x^2 = 9x$ (2)

d) Solve for x : $x^2 + x - 12 > 0$ (2)

e) Simplify fully $e^{\ln(2x-1)}$ (1)

f) Solve for x : $|2x-3| \leq 10$ (2)

g) Differentiate $\cos(3x^2)$ (2)

Question 2 **(Begin a new Booklet)**

a) A fair coin is tossed three times. What is the probability that exactly one tail is obtained? (2)

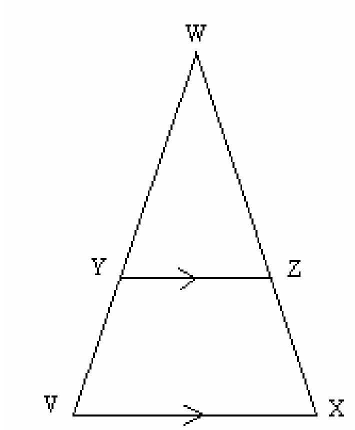
b) Differentiate:

i) $x \ln x$ (2)

ii) $\dfrac{2x+1}{e^x}$ (2)

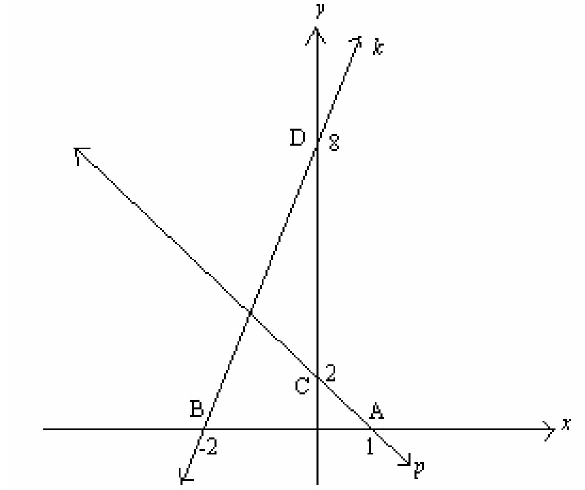
c) Solve the equation $\sqrt{5x+2} = 7$ (2)

d) In triangle WXV, YZ = 12 cm, VX = 16 cm, WX = 8 cm and YZ ∥ VX. Prove that ΔWZY is similar to ΔWXV and find the length of WZ. (4)



NOT TO SCALE

Question 3 **(Begin a new Booklet)**



In the diagram above, the line p cuts the y axis at C (0, 2) and the x axis at A (1, 0) while the line k cuts the y axis at D (0, 8) and the x axis at B (-2, 0).

a) Find the equation of the line k and the line p. (2)

b) Find the coordinates of the point Q, where lines k and p intersect. (2)

c) Write the equation of a line, q, passing through Q and perpendicular to the x axis. (2)

d) Find the area enclosed by the lines p and q and the x axis. (2)

e) Find the equation of the line t through the point A (1, 0) and parallel to the line k. (2)

f) Find the perpendicular distance between the line t and the line k. (2)

Question 4 **(Begin a new Booklet)**

a) P (x, y) moves so that its distance from M (3, 0) is always twice its distance from the point N (0, 3).

i) Show that the equation of the locus of all points P (x, y) is

$$x^2 + 2x + y^2 - 8y + 9 = 0$$ (4)

ii) Give a geometrical description of the locus. (2)

b) Find the equation of the curve $y = f(x)$ which satisfies the conditions:

- $f'(x) = e^x + b$
- (0, 7) lies on the curve, and
- the slope of the tangent at $x = 0$ is 3. (4)

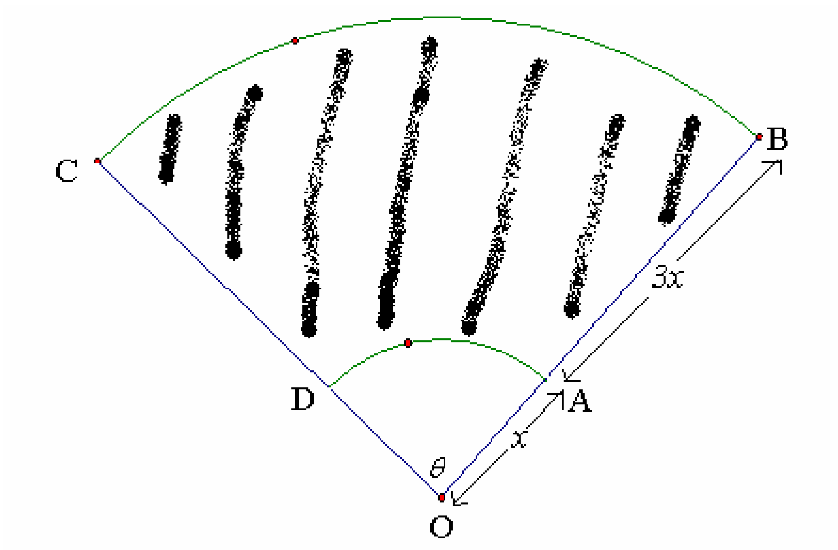
c) Find i) $\int \sec^2(3x)dx$ (1)

ii) $\int \frac{5}{3x+2} dx$ (1)

Question 5 **(Begin a new Booklet)**

a) Solve $4-4\cos 2x=2$ for $0\le x\le 2\pi$ (2)

b) In the diagram below, a cars windscreen wiper blade sweeps across the region ABCD, where BC and AD are the arcs of circles with centre O. The intervals OA and AB are x cm and $3x$ cm respectively, with $\angle BOC = \theta$. The perimeter of the shaded region ABCD is 240 cm.



i) Find the angle θ, in terms of x. Show that $\theta=\dfrac{240-6x}{5x}$ (1)

ii) Show that the area ABCD is: $9x(40-x)$ cm^2 (1)

iii) Find the maximum area of the shaded region. (2)

c) Prove $\dfrac{\sin^3\theta}{\cos\theta}+\sin\theta\cos\theta=\tan\theta$. (3)

d) Differentiate $y=x\sin x+\cos x$. Hence, find in exact form, $\int_0^{\frac{\pi}{2}} x\cos x\,dx$. (3)

Question 6 next page….

Question 6 **(Begin a new Booklet)**

a) A parabola has the equation $x^2 - 6x - 6y - 3 = 0$. For this parabola, find:

i) the focal length,

ii) the coordinates of the vertex,

iii) the coordinates of the focus,

iv) the equation of the directrix (5)

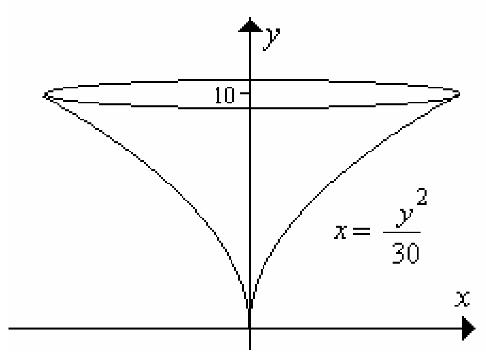
b) α and β are the roots of $(2m-1)x^2 + (1+m)x + 1 = 0$. Find m if $\alpha + \beta = 0$. (2)

c) The table below shows points on a continuous curve $y = f(x)$. Use the trapezoidal rule to find the approximate value, to 2 decimal places, of $\int_3^{3.4} f(x)dx$. (2)

x	3	3.2	3.4
$f(x)$	7.19	7.62	8.41

d) A glass shape is obtained by rotating part of the parabola $x = \frac{y^2}{30}$ about the y axis as shown. The glass is 10 cm deep.

Find the volume of liquid, to 1 decimal place, which the glass will hold. (3)



Question 7 next page….

Question 7 (Begin a new Booklet)

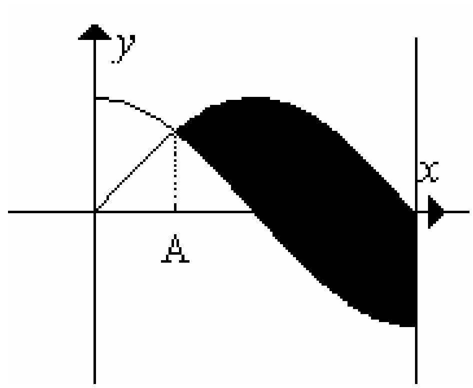
a) The function $y = x^3 - 3x^2 - 9x + 1$ is defined in the domain $-4 \le x \le 5$.

i) Find the coordinates of any turning points and determine their nature.

ii) Find the coordinates of any points of inflexion.

iii) Determine the minimum value of the function y in the domain $-4 \le x \le 5$. (7)

b)



The diagram above shows $y = \sin x$ and $y = \cos x$ for the domain $0 \le x \le \pi$.

i) Show that the value of A, the x coordinate for the point of intersection of the two curves, is $\frac{\pi}{4}$. (2)

ii) Find the size of the shaded area in exact form with a rational denominator in simplest form. (3)

Question 8 next page…..

Question 8 (Begin a new Booklet)

a) The mass M of radioactive substance present after t years is given by $M = 12e^{-kt}$ where k is a positive constant. After 150 years, the mass reduced to 4 kg.

i) What was the original mass? (1)

ii) Find the value of k, to 4 decimal places. (1)

iii) What amount of the substance would remain after a period of 500 years, to 2 significant figures? (1)

iv) How long, to the nearest year, would it take for the original mass to reduce to 6 kg? (2)

b) A 12 cm bamboo plant is put into a garden and its growth is recorded. It grows 5cm every week. In what week will its height reach 167 cm? (2)

c) The sum of the first n terms of a series is given by $S_n = 2^n + n^2$.
Find the 15th term. (2)

d) A geometric series is generated by the rule $\sum_{n=1}^{8} 16\left(\frac{3}{2}\right)^{n-1}$.

i) Write out the first three terms of the series. (1)

ii) Find the exact value of the sum of this series. (2)

Question 9 next page…..

Question 9 (Begin a new Booklet)

a) A closed water tank in the shape of a right cylinder is to be constructed with a surface area of 64π cm^2. The height of the cylinder is h cm and the base radius is r cm.

i) Show that the height of the water tank, in terms of r, is given by (2)

$$h = \frac{32}{r} - r$$

ii) Show that the volume, V, that can be contained in the tank is given by (1)

$$V = 32\pi r - \pi r^3$$

iii) Find the radius r cm which will give the cylinder its greatest possible volume. Justify your answer. (3)

b) Oz Challenge is a game played with two different coloured dice: one gold and the other blue.

The six faces of the blue die are numbered: 5, 7, 9, 10, 11, 13
The six faces of the gold die are numbered: 1, 4, 6, 8, 12, 14

The player wins if the number on the gold die is larger than the number of the blue die.

i) Calculate the probability of the player winning a game. (2)

ii) Calculate the probability that the player wins at least once in 2 successive games. (2)

c) Solve $\log_e(x^2 - x) = \log_e 2 + \log_e(3x + 4)$ (2)

Question 10 next page……

Question 10 **(Begin a new Booklet)**

a) Daniel borrows \$8 000 from his father to pay for his World Cup Football trip and tickets. They agree that Daniel should pay interest of 1.5% every month and that he should agree to pay his father back an instalment every month.

i) Letting \$A be the amount owing after n months, and \$T be the value of each monthly instalment, derive an expression, involving T, for the amount owing after 12 months. (1)

ii) Hence, find the value of T, to the nearest dollar, if he repays the loan after two years. (5)

b) A particle moves on a horizontal line so that its displacement x cm to the right of the origin at time t seconds is $x = t\sin t$.

i) Find expressions for the velocity and acceleration of the particle. (2)

ii) Find the exact velocity of the particle at time $t = \frac{\pi}{4}$. (1)

iii) What effect does the acceleration have on the velocity of the particle at $t = \frac{\pi}{4}$. (1)

iv) After the particle leaves the origin, is the particle ever at rest? Give reasons for your answer. (2)

The End

Q1

a) $\sqrt[3]{\frac{(9.2)^2}{\pi}} = 2.9978\ldots$

$= 3.0$ (2 sig fig) ✓

b) $\frac{6a^2-2ab}{9a^2-b^2} = \frac{2a(3a-b)}{(3a-b)(3a+b)}$ ✓ ✓

$= \frac{2a}{3a+b}$

c) $18x^2 = 9x$

$18x^2 - 9x = 0$

$9x(2x-1) = 0$

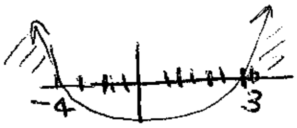
$x = 0$ ✓ or $2x - 1 = 0$

$2x = 1$

$x = \frac{1}{2}$ ✓

d) $x^2 + x - 12 > 0$

$(x+4)(x-3) > 0$



$x < -4$ ✓ or $x > 3$ ✓

e) $e^{\ln(2x-1)} = 2x - 1$ ✓

f) $|2x-3| \leq 10$

$-10 \leq 2x - 3 \leq 10$

$-7 \leq 2x \leq 13$

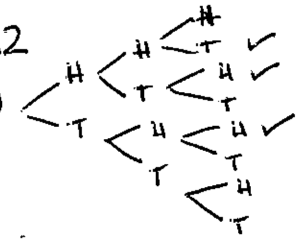
$-\frac{7}{2}$ ✓ $\leq x \leq \frac{13}{2}$ ✓

g) $y = \cos(3x^2)$

$\frac{dy}{dx} = -6x$ ✓ $\sin(3x^2)$ ✓

Q2

a)



$P(=1T) = \frac{3}{8}$ ✓ ✓

b) i) $y = x \ln x$ $\quad u = x \quad v = \ln x$

$\frac{du}{dx} = 1 \quad \frac{dv}{dx} = \frac{1}{x}$

$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

$= \ln x . 1 + x . \frac{1}{x}$ ✓✓

$= \ln x + 1$

ii) $y = \frac{2x+1}{e^x}$ $\quad u = 2x+1 \quad v = e^x$

$\frac{du}{dx} = 2 \quad \frac{dv}{dx} = e^x$

$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$= \frac{e^x . 2 - (2x+1)e^x}{(e^x)^2}$

$= \frac{e^x(2 - 2x - 1)}{(e^x)^2}$

$= \frac{1-2x}{e^x}$ ✓✓

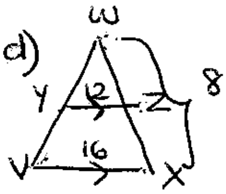
c) $\sqrt{5x+2} = 7$

$5x + 2 = 49$ ✓

$5x = 47$

$x = \frac{47}{5}$ ✓

d)



In ΔWYZ & ΔWVX ✓✓✓

$\angle W = \angle W$ (common)

$\angle WYZ = \angle WVX$ (alt. $\angle$'s equal YZ ∥ VX)

$\angle WZY = \angle WXV$ (alt $\angle$'s equal YZ ∥ VX)

∴ ΔWYZ similar ΔWVX as they are equiangular.

$\frac{12}{16} = \frac{x}{8}$

$x = 6$ cm ✓

Q3.

a) $m_k = \frac{8-0}{0+2}$
$= 4$

$4 = \frac{y-8}{x}$
$4x = y - 8$ ✓
"k" $y = 4x + 8$

$m_p = \frac{2-0}{0-1}$
$= -2$
$-2 = \frac{y-2}{x}$
$-2x = y - 2$
"p" $y = -2x + 2$ ✓

b) $4x + 8 = -2x + 2$ ✓
$6x = -6$
$x = -1$

$y = -4 + 8$
$y = 4$
$\therefore Q \ (-1, 4)$ ✓

c) $x = -1$

d)

$A = \frac{1}{2}bh$
$= \frac{1}{2} \times 4 \times 2$
$= 4 \text{ units}^2$ ✓ ✓

e) $m_k = 4$ $\quad m_t = 4$ ✓ if parallel to k.

$\therefore 4 = \frac{y-0}{x-1}$
$4x - 4 = y$
$y = 4x - 4$ ✓

f) $4x - y + 8 = 0 \quad (1, 0)$ $\quad a = 4, \ b = -1, \ c = 8 \quad x = 1, \ y = 0$

$d = \left| \frac{ax + by + c}{\sqrt{(a^2 + b^2)}} \right|$

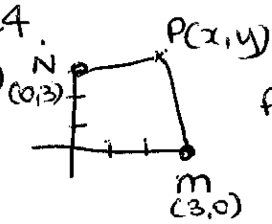
$= \left| \frac{4 \times 1 + -1 \times 0 + 8}{\sqrt{(4^2 + -1^2)}} \right|$ ✓

$= \left| \frac{12}{\sqrt{17}} \right|$

$= \frac{12}{\sqrt{17}}$ units ✓

Q4.

a) i)



$2PN = PM$ ✓

$PN = \sqrt{(x-0)^2 + (y-3)^2}$ ✓
$PM = \sqrt{(x-3)^2 + (y-0)^2}$

$2\sqrt{x^2 + (y-3)^2} = \sqrt{(x-3)^2 + y^2}$ ✓
$4(x^2 + y^2 - 6y + 9) = (x^2 - 6x + 9) + y^2$
$4x^2 + 4y^2 - 24y + 36 = x^2 - 6x + 9 + y^2$
$3x^2 + 6x + 3y^2 - 24y + 27 = 0$
$(\div 3)$ $\quad x^2 + 2x + y^2 - 8y + 9 = 0$ ✓

ii) $x^2 + 2x + \left(\frac{2}{2}\right)^2 + y^2 - 8y + \left(\frac{8}{2}\right)^2 = -9 + 1^2 + 4^2$
$(x+1)^2 + (y-4)^2 = 8$

Circle centre $(-1, 4)$ ✓ $\quad r = \sqrt{8}$
$= 2\sqrt{2}$ units ✓

b) $f'(x) = e^x + b$
$f'(x) = 3$ when $x = 0$
$e^0 + b = 3$
$1 + b = 3$ ✓
$b = 2$
$f'(x) = e^x + 2$ ✓
$f(x) = e^x + 2x + c$ ✓
when $x = 0$, $f(x) = 7$
$e^0 + 0 + c = 7$
$c = 6$ ✓
$\therefore f(x) = e^x + 2x + 6$

c) i) $\int \sec^2 3x \, dx = \frac{1}{3}\tan 3x + c$ ✓

ii) $\int \frac{5}{3x+2} dx = \frac{5}{3} \int \frac{3}{3x+2} dx$ ✓
$= \frac{5}{3} \ln(3x+2) + c$

Q5.

a) $4 - 4\cos 2x = 2$

$4(1-\cos 2x) = 2$

$1 - \cos 2x = \frac{1}{2}$

$+\cos 2x = +\frac{1}{2}$

$2x = \frac{\pi}{3}$ $2x = \frac{5\pi}{3}$ $2x = \frac{7\pi}{3}$ $2x = \frac{11\pi}{3}$

$x = \frac{\pi}{6}$ $x = \frac{5\pi}{6}$ $x = \frac{7\pi}{6}$ $x = \frac{11\pi}{6}$

$\therefore x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ ✓ ✓

b) i) Small $l = r\theta$ $= 3x\theta$ large $l = r\theta$ $= 4x\theta$

$\therefore$ Perimeter $= 3x + 3x + x\theta + 4x\theta$

$240 = 6x + 5x\theta$

$5x\theta = 240 - 6x$

$\theta = \frac{240-6x}{5x}$ ✓

ii) Area $= \frac{1}{2} \cdot 16x^2\theta - \frac{1}{2}x^2\theta$

$= \frac{1}{2} 15x^2\theta$

$= \frac{1}{2} 15x^2 \left(\frac{240-6x}{5x}\right)$

$= 3x(120-3x)$

$= 9x(40-x)$ ✓

$= 360x - 9x^2$

iii) max area occurs when $\frac{dA}{dx} = 0$, $x = 20$

$\frac{dA}{dx} = 360 - 18x$

$360 - 18x = 0$

$18x = 360$

$x = 20$ ✓

$= 9 \times 20 (40-20)$

$= 180(20)$

$= 3600 \text{ cm}^2$ ✓

c) LHS $\frac{\sin^3\theta}{\cos\theta} + \sin\theta\cos\theta$

$= \frac{\sin^3\theta + \sin\theta\cos^2\theta}{\cos\theta}$ ✓

$= \frac{\sin\theta(\sin^2\theta + \cos^2\theta)}{\cos\theta}$ ✓

$= \frac{\sin\theta}{\cos\theta}$ ✓

$= \tan\theta$ = RHS.

d) $y = x\sin x + \cos x$ $u = x$ $v = \sin x$ $\frac{du}{dx} = 1$ $\frac{dv}{dx} = \cos x$

$\frac{dy}{dx} = \sin x + x\cos x + (-\sin x)$

$= x\cos x$ ✓✓

$\int_0^{\frac{\pi}{2}} x\cos x\,dx = \left[x\sin x + \cos x\right]_0^{\frac{\pi}{2}}$

$= \left(\frac{\pi}{2}\sin\frac{\pi}{2} + \cos\frac{\pi}{2}\right) - (0 + \cos 0)$

$= \frac{\pi}{2} - 1$ ✓

Q6

a) $x^2 - 6x - 6y - 3 = 0$

$(x^2 - 6x + (\frac{6}{2})^2) = 6y + 3 + 9$

$(x-3)^2 = 6(y+2)$ ✓

i) $4a = 6$ $a = \frac{3}{2}$ Focal length $= \frac{3}{2}$ ✓

ii) vertex $(3, -2)$ ✓

iii) focus $(3, -\frac{1}{2})$ ✓

iv) directrix $y = -3\frac{1}{2}$ ✓

b) $\alpha + \beta = \frac{-b}{a}$ but $\alpha + \beta = 0$

$0 = \frac{-(1+m)}{(2m-1)}$ ✓ but $m \neq \frac{1}{2}$

$-m - 1 = 0$

$m = -1$ ✓

c) $A = \frac{0.2}{2}\{7.19 + 8.41 + 2 \times 7.62\}$ ✓

$= 3.084 \text{ units}^2$ ✓

d) $V = \pi\int_a^b x^2\,dy$

$= \pi\int_0^{10}\left(\frac{y^2}{30}\right)^2 dy$ ✓

$= \pi\int_0^{10}\frac{y^4}{900}\,dy$

$= \pi\left[\frac{y^5}{4500}\right]_0^{10}$ ✓

$= \pi\left[\frac{10^5}{4500}\right]$

$= 69.81\ldots$

$= 69.8 \text{ cm}^3$ ✓

Q7

a) $y = x^3 - 3x^2 - 9x + 1$

i) $\frac{dy}{dx} = 3x^2 - 6x - 9$ ✓

St pts occur $\frac{dy}{dx} = 0$

$3x^2 - 6x - 9 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$ ✓

$x = 3$ or $x = -1$

x	-1^-	-1	-1^+
$\frac{dy}{dx}$	+	0	−

x	3^-	3	3^+
$\frac{dy}{dx}$	−	0	+

$(-1, 6)$ maximum $(3, -26)$ minimum ✓

ii) $\frac{d^2y}{dx^2} = 6x - 6$ ✓ pt inflection occurs $\frac{d^2y}{dx^2} = 0$

$6x - 6 = 0$

$x - 1 = 0$

$x = 1$

x	1^-	1	1^+
$\frac{d^2y}{dx^2}$	−	0	+

∴ $(1, -10)$ pt inflection ✓

iii) when $x = -4$ $y = -75$ ✓

$x = 5$ $y = 6$ ✓

∴ min value $-4 \leq x \leq 5$

is $y = -75$ when $x = -4$

b) i) $\sin x = \cos x$

$\frac{\sin x}{\cos x} = 1$ ✓

$\tan x = 1$ ✓

$x = \frac{\pi}{4}$ $0 \leq x \leq \pi$

ii) $\int_{\frac{\pi}{4}}^{\pi} \sin x \, dx - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x \, dx + \left| \int_{\frac{\pi}{2}}^{\pi} \cos x \, dx \right|$ ✓

$= \left[-\cos x\right]_{\frac{\pi}{4}}^{\pi} - \left[\sin x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} + \left| \left[\sin x\right]_{\frac{\pi}{2}}^{\pi} \right|$

$= \left[-\cos\pi + \cos\frac{\pi}{4}\right] - \left[\sin\frac{\pi}{2} - \sin\frac{\pi}{4}\right] + \left|\left[\sin\pi - \sin\frac{\pi}{2}\right]\right|$ ✓

$= \left(1 + \frac{1}{\sqrt{2}}\right) - \left(1 - \frac{1}{\sqrt{2}}\right) + |-1|$

$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + 1$

$= 1 + \frac{2}{\sqrt{2}}$

$= \frac{\sqrt{2}+2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{2 + 2\sqrt{2}}{2}$

$= 1 + \sqrt{2}$ units^2 ✓

Q8

a) $M = 12e^{-kt}$

i) when $t = 0$

$M = 12e^0$ ✓

$= 12$ kg ∴ initial mass = 12 kg

ii) $M = 12e^{-kt}$ when $M = 4$, $t = 150$

$4 = 12e^{-k \times 150}$

$\frac{1}{3} = e^{-150k}$

$\ln\left(\frac{1}{3}\right) = -150k$

$k = \frac{\ln\left(\frac{1}{3}\right)}{-150}$

$= 0.007324$

$= 0.0073$ (to 4dp) ✓

iii) $M = 12e^{-0.0073t}$

when $t = 500$

$M = 12e^{-0.0073 \times 500}$

$= 0.31189..$

$= 0.31$ kg (to 2 sig figs) ✓

iv) $M = 12e^{-0.0073t}$ when $M = 6$

$6 = 12e^{-0.0073t}$ ✓

$\frac{1}{2} = e^{-0.0073t}$

$\ln\left(\frac{1}{2}\right) = -0.0073t$

$t = \frac{\ln\left(\frac{1}{2}\right)}{-0.0073}$

$= 94.95$ ✓

$= 95$ yrs (nearest yr).

b) $12 + 17 + 22 + \ldots + 167$ $a = 12, d = 5, T_n = 167$

$T_n = a + (n-1)d$

$167 = 12 + (n-1)5$ ✓

$155 = 5n - 5$

$5n = 160$

$n = 32$ ∴ 32nd week ✓

c) $S_n = 2^n + n^2$

$S_{15} = 2^{15} + 15^2$

$= 32993$ ½

$S_{14} = 2^{14} + 14^2$

$= 16580$ ½

$T_{15} = S_{15} - S_{14}$

$= 32993 - 16580$

$T_{15} = 16413$ ✓

d) $\sum^{8} 16\left(\frac{3}{2}\right)^{n-1}$

i) $n = 1$

$n = 1$ $16\left(\frac{3}{2}\right)^0 = 16$

$n = 2$ $16\left(\frac{3}{2}\right)^1 = 24$

$n = 3$ $16\left(\frac{3}{2}\right)^2 = 36$

1st term $a = 16$

2nd term $= 24$

3rd term $= 36$ ✓

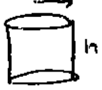
ii) $S_8 = \frac{a(r^n - 1)}{r - 1}$

$= \frac{16\left(\left(\frac{3}{2}\right)^8 - 1\right)}{\frac{3}{2} - 1}$ ✓

$= \frac{32\left(\frac{6561}{256} - 1\right)}{}$

$= 788\frac{1}{8}$ ✓

Q9.

a) 

$SA = 2\pi r^2 + 2\pi r h$

$2\pi r^2 + 2\pi r h = 64\pi$ ✓

$2\pi r(r+h) = 64\pi$

$r(r+h) = 32$

$r + h = \frac{32}{r}$

$h = \frac{32}{r} - r$ ✓

ii) $V = \pi r^2 h$

$= \pi r^2 \left(\frac{32}{r} - r\right)$

$= 32\pi r - \pi r^3$ ✓

iii) $\frac{dV}{dr} = 32\pi - 3\pi r^2$

max volume occurs $\frac{dV}{dr} = 0$ ✓

$32\pi - 3\pi r^2 = 0$

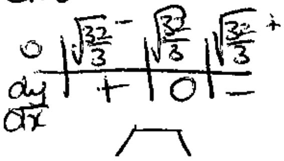
$3\pi r^2 = 32\pi$

$r^2 = \frac{32\pi}{3\pi}$ ✓

$= \frac{32}{3}$

$r = \pm\sqrt{\frac{32}{3}}$ cm

as $r > 0$ $r = \sqrt{\frac{32}{3}}$ cm



✓

∴ max. value

b)

Blue	Gold
5	6, 8, 12, 14
7	8, 12, 14
9	12, 14
10	12, 14
11	12, 14
13	14

$n(\text{win}) = 14$ ✓

i) $P(\text{win}) = \frac{14}{36}$

$= \frac{7}{18}$ ✓

$P(\text{loss}) = \frac{11}{18}$

ii) $P(\text{at least once})$

$= 1 - P(2L)$

$= 1 - \left(\frac{11}{18} \times \frac{11}{18}\right)$

$= 1 - \frac{121}{324}$

$= \frac{203}{324}$

c) $\log_e(x^2 - x) = \log_e 2 + \log_e(3x+4)$

$\log_e(x^2 - x) = \log_e 2(3x+4)$

$x^2 - x = 6x + 8$ ✓

$x^2 - 7x - 8 = 0$

$(x-8)(x+1) = 0$

$x - 8 = 0$ or $x + 1 = 0$

$x = 8$ $x = -1$ ✓

Q10

a) 1st Mth $= 8000 \times 1.015 - T$

i) 2nd Mth $= (8000 \times 1.015 - T)1.015 - T$

$= 8000 \times 1.015 \times 1.015 - T1.015 - T$

3rd Mth $= [\text{2nd Mth}] \times 1.015 - T$

$= 8000 \times 1.015^3 - 1.015^2 T - 1.015T - T$ ✓

⋮

$A_n = 8000 \times 1.015^n - 1.015^{n-1} T \ldots - 1.015T - T$

ii) $A_{24} = 8000 \times 1.015^{24} - 1.015^{23} T \ldots - 1.015T - T$

$= 8000 \times 1.015^{24} - T(1.015^{23} \ldots + 1.015 + 1)$

But $A_{24} = 0$

$8000 \times 1.015^{24} = T(1 + 1.015 + \ldots 1.015^{23})$

$T = \frac{8000 \times 1.015^{24}}{1 + 1.015 + \ldots 1.015^{23}}$ ✓

GP $a = 1$, $r = 1.015$ $n = 24$ ✓

$S_n = \frac{a(r^n - 1)}{r - 1}$

$= \frac{1(1.015^{24} - 1)}{1.015 - 1}$

$= 28.63\ldots$ ✓

$T = \frac{8000 \times 1.015^{24}}{28.63\ldots}$ ✓

$= \$399.39$ every month ✓

b) $x = t\sin t$

i) $\frac{dx}{dt} = v = \sin t + t\cos t$ ✓

$u = t$ $v = \sin t$, $\frac{du}{dt} = 1$ $\frac{dv}{dt} = \cos t$

$\frac{dv}{dt} = a = \cos t + \cos t - t\sin t$

$u = t$ $v = \cos t$, $\frac{du}{dt} = 1$ $\frac{dv}{dt} = -\sin t$

$a = 2\cos t - t\sin t$ ✓

ii) when $t = \frac{\pi}{4}$ $v = \sin\frac{\pi}{4} + \frac{\pi}{4}\cos\frac{\pi}{4}$

$= \frac{1}{\sqrt{2}} + \frac{\pi}{4} \times \frac{1}{\sqrt{2}}$ ✓

$= \frac{1}{\sqrt{2}} + \frac{\pi}{4\sqrt{2}}$ cm/s

iii) $a = 2\cos\frac{\pi}{4} - \frac{\pi}{4}\sin\frac{\pi}{4}$

$= 2 \times \frac{1}{\sqrt{2}} - \frac{\pi}{4} \times \frac{1}{\sqrt{2}}$

$= \frac{2}{\sqrt{2}} - \frac{\pi}{4\sqrt{2}}$ cm/s² ✓

as acceleration is positive, the particle is speeding up the particle. ✓

iv) at rest $v = 0$ ie $\sin t + t\cos t = 0$

Sketch $y = \sin t + t\cos t$

we can see that it will be at rest again

at this point the y values added = 0